KINETIC THEORY OF PLASMA WAVES - Part III: Inhomogeneous plasma

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ABSTRACT

Given the extent of the subject, the present text merely provides a list of topics, striking features and references relevant to the kinetic theory of high-frequency wave propagation and absorption in inhomogeneous plasmas. The discussion focuses on tokamak geometry.

I. INTRODUCTION

Real plasmas are inhomogeneous. A large number of theories and numerical models have been developed to describe the effects of inhomogeneities on wave propagation and absorption, at various levels of complexity.

'Mild' inhomogeneities, in the context of geometric optics, allow a straightforward transposition of the theoretical results obtained for homogeneous plasma waves, using the local properties of the medium at each point of space. This asymptotic theory of wave propagation leads to powerful analytical and numerical ray-tracing methods, particularly important for the modelling of electron cyclotron and lower hybrid heating and current drive. This is addressed in Section II.

'Strong' inhomogeneities, or situations where geometric optics is not a valid approximation, require a so-called 'full-wave' treatment, in which the various wave modes can strongly interact with each other, as well as with the boundaries of the system. This case leads to much more complex theoretical developments and numerical investigations, for which the methods of predilection are finite difference and finite element discretizations of the wave equation. The Maxwell system of equations (I.35)[1] is treated as a boundary value problem. Ion cyclotron resonance heating requires this type of approach in many situations (the local analysis of geometric optics also provides invaluable physical insight in this case, but its ability to make quantitative predictions is limited, sometimes severely).

Section III presents some important features of wave propagation, wave absorption and quasilinear heating in tokamak geometry.

It is obviously impossible to give credit to all relevant work here. Many additional references will be found in the references themselves.

II. GEOMETRIC OPTICS [1-9]

A. The geometric optics, or Wentzel-Kramers-Brillouin-Jeffreys (WKBJ) approximation (see Chapter 4 of [1], [2]), assumes that the wave pattern can be described as a superposition of noninteracting modes, their wavevector **k** satisfying the local dispersion relation introduced in Part II. Its condition of validity is the following:

$$\|\nabla \mathbf{k}\| << k^2 \qquad (1),$$

which states that the wavelength must not change much over a wavelength.

The approximation breaks down in a number of situations:

- In the presence of strong wave focussing and / or reflection;
- In mode conversion regions, that is regions where the wavelengths of two distinct branches of the dispersion become comparable and the modes interact with each other. Wave cutoffs (where **k** or one of its components vanishes) fall into this category;
- Inside evanescent regions.

Ray tracing is a powerful modelling tool, providing much insight on wave propagation. It is essential for the simulation of lower hybrid and electron cyclotron heating and current drive, for which the wavelengths are much smaller than the machine dimensions (finite difference or finite element discretizations would require huge meshes, and would be totally impractical in 2 and 3 dimensions). Ray tracing has also been used for the simulation of ion cyclotron resonance heating, where its application to propagation of the fast magnetosonic wave is restricted to large machines, and is nowadays more or less superseded by full-wave codes. It is still of high interest in heating scenarios where mode conversion to a Bernstein wave is

[1] This notation indicates reference to an equation of Part I.

important. The feasibility of its combined use with full-wave methods has been demonstrated [10, 11].

With ray tracing, propagation is reduced to an ensemble of independent initial value problems. This is in striking contrast with the boundary value problem described by Maxwell's equations. The initial conditions give the starting wavevector and power density ascribed to the wave mode at each point of a reference surface; these are obtained by means of auxiliary models describing the wave launcher (antenna, waveguide) and its surroundings (machine wall, vacuum and low density plasma).

B. The ray equations:

- The dispersion relation has been presented in Part II. Here a dependence on position appears:

$$\det \Lambda \equiv D(\omega, \mathbf{k}, \mathbf{r}) = 0 \quad (2)$$

- The ray trajectory and the evolution of the wavevector are given by

$$\dot{\mathbf{r}} = -\left.\frac{\partial D/\partial \mathbf{k}}{\partial D/\partial \omega}\right|_{D=0} = \mathbf{v_g}\,, \quad \dot{\mathbf{k}} = \left.\frac{\partial D/\partial \mathbf{r}}{\partial D/\partial \omega}\right|_{D=0} \quad (3)$$

where $\mathbf{v}_g$ is the group velocity. The rate of power transfer from the wave mode to the plasma is given by [7]

$$\dot{P}_F = 2\left.\frac{\partial D/\partial k_\perp^2}{\partial D/\partial \omega}\right|_{D=0} im\, k_\perp^2 \quad (4)$$

Note that, in regions of strong wave damping, the right-hand sides of equations (3, 4) become complex and their interpretation may not be obvious. On this topic, see e.g. [8, 9].

III. WAVE HEATING OF TOKAMAK PLASMAS [12-41]

A. In tokamak geometry, the toroidal curvature and the poloidal magnetic field introduce specific effects which have an strong influence on wave propagation and absorption:

1. The 1/R dependence of the equilibrium magnetic field, hence of ω_c, makes the resonance condition (I.1) strongly depend on position and gives rise to narrow cyclotron resonance layers (see Fig.1) Several cyclotron layers may coexist due to

- Presence of ions with different charge-to-mass ratios;
- In large machines, presence of more than one harmonic of the same ion species;
- Simultaneous use of several hf antennae powered at different frequencies.

The characteristic half-width of the resonance layer is given by

$$\frac{\Delta R}{R} \sim \left|\frac{k_{//} v_T}{p\,\omega_c}\right| \quad (5)$$

where v_T is the thermal velocity for Maxwellian populations, or a characteristic maximum velocity for other distribution functions. In typical ICRH experiments, this width is a few percent of the major radius for the bulk ions, but may exceed 30% for MeV ions. In contrast, inhomogeneity in the Landau resonance condition (p=0) is much milder, as it only results from spatial variations of the Doppler shift $k_{//}v_{//}$.

2. Particle orbits are remarkably different from their homogeneous plasma counterparts [I.4]: the guiding centre (gc) motion along the magnetic field lines can be highly nonuniform, there are passing and trapped orbits, and the radial gc drifts can be considerable. In sufficient numbers, energetic particles, such as MeV ICRH-accelerated ions or fusion alphas, have a strong influence on wave propagation and absorption. Their large Larmor radii and large radial gc drifts are important features, particularly challenging to theoretical and numerical investigations.

Each particle experiences a time-dependent magnetic induction, $\dot{B}_0 \approx v_{//}\nabla_{//}B_0$ (6),
and its Larmor gyration at the cyclotron frequency is modulated along the orbit. As a consequence, it only spends a small fraction of its trajectory in cyclotron resonance. Far from the zeros of equation (6), the particle transits through cyclotron resonance in a characteristic time

$$\tau_s \sim \sqrt{2\pi/|\,p\,\dot{\omega}_{cs}\,|} \quad (7)$$

(that is the time required for the parallel inhomogeneity to modify the gyrophase by π)

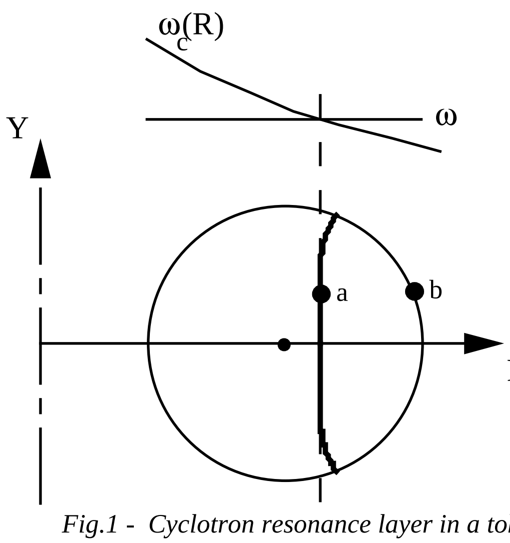

Fig.1 - Cyclotron resonance layer in a tokamak (schematic). The upper part of the figure sketches the 1/R dependence of the cyclotron frequency.

3. Various orbit / resonance layer configurations contribute to the overall picture: the poloidal projection of the gc motion is periodic, hence most particles repeatedly transit through the resonance layer, typically twice per period for passing and four times for trapped orbits. (Comparing various orbits through points a and b on Fig.1,

one easily convinces oneself of the tremendous difference introduced by the poloidal field.) Some particles are too deeply trapped to reach resonance, and do not absorb any hf power through cyclotron interactions. The intermediate orbit configurations, which graze resonance quasi-tangentially, either near the banana tips or near midplane crossing, are also very important. There, a longer time is spent in a single tangent resonance, or two correlated resonance crossings closely follow each other.

4. Phase decorrelation

Cyclotron heating requires randomization of the relative phase between wave and particle gyromotion after each resonance crossing. Indeed, if phase memory was maintained for ever, the successive resonant 'kicks' received by the particle would interfere with each other and cancel on average ('superadiabatic' regime, see [1], §16-9 and §17-14). Various mechanisms, such as nonlinear effects [13-15] or collisions [16-18], can provide this phase decorrelation. Collisional decorrelation is in general extremely efficient for cyclotron interactions in tokamaks. This effect is another consequence of the parallel gradient in the magnetic field [17]. The parallel gradient of the hf field (cf. $k_{//}$) plays a similar role, usually weaker for cyclotron interactions but very important for the Landau interaction [16].

One should be aware that superadiabatic orbits do occur, in particular at high energies where collisions are rare; correlated resonance crossings occur in the particular quasi-tangent resonance configurations mentioned above.

B. Toroidal geometry is also responsible for striking effects on the evolution of the distribution functions under wave heating, described by the bounce-averaged quasilinear Fokker-Planck equation mentioned in equation (I.39):

1. Cyclotron heating mainly increases the perpendicular components of the velocity, and thus tends to transfer the resonant particles onto trapped orbits. This trapping may progressively increase until the banana tips graze the resonance layer. Collisional relaxation competes with this mechanism, and the resulting steady-state distribution functions display a typical 'rabbit-ear' shape, with accumulation of high-energy particles on trapped orbits grazing resonance with their tips [33], [1] Fig.18-2.

2. Asymmetric antenna toroidal wavenumber spectra induce in- or outward radial drifts of the banana orbits, see [1] §17-4 and [39-41].

C. Resonances in inhomogeneous plasmas:

In Part I, we noted different forms for the resonance condition in homogeneous and inhomogeneous plasmas, equations (I.1) and (I.31'). More explicitly, the latter is

$$\omega = p\left\langle \omega_c \right\rangle_b + n\left\langle \dot{\varphi} \right\rangle_b + j\omega_b \qquad (9)$$

This is a global condition for each gc orbit, where $< >_b$ indicates time-averaging over one poloidal period of the gc motion (i.e. over the 'bounce' time τ_b). p is the cyclotron harmonic index, n the toroidal Fourier mode index of the hf perturbation and j the bounce harmonic index. In principle, the plasma hf dielectric response and the quasilinear diffusion operator include contributions from all integer triplets (p, n, j), but p usually takes a few significant values determined by the heating scenario under investigation, and the width of the n spectrum is determined by the hf antenna. $\dot{\varphi}$ is the toroidal angular velocity of the gc, and $\omega_b = 2\pi/\tau_b$ the bounce frequency, typically much smaller than ω and ω_c.

For the cyclotron interactions (i.e. p nonzero), the bounce harmonic spectrum typically contains a very large number of significant terms, see Fig.5 of [25]. The number of terms is much smaller for some orbits (confined close to the magnetic axis or deeply trapped), and for the Landau interaction (p=0).

When a finite decorrelation time t_L is introduced in the theory, one finds that the number of significant terms in the bounce harmonic series scales as τ_b/t_L: if decorrelation is rapid, many terms contribute and the individual resonances of equation (9) are actually smeared; in the opposite limit, correlation effects may take place over several bounce times, there only are a few terms in the bounce harmonic series and individual wave-cyclotron-bounce resonances emerge. This strongly nonlocal regime is more likely to occur either at high energies, or for orbits of very small poloidal extent, or for p=0.

Finally, note that the local expression of the resonance condition (I.1) is recovered in the asymptotic limit of *strong* inhomogeneity along the orbits (this is generally the case for cyclotron interactions). For a detailed discussion, see [25]; the explanation of this apparently paradoxical result is found in the highly oscillatory trajectory integral (I.30), which can be evaluated as a sum of saddle point contributions [2] in this limit. The saddle point locations along the orbit satisfy the equation

$$\omega - p\left\langle \omega_c \right\rangle_b - n\left\langle \dot{\varphi} \right\rangle_b - j\omega_b = \omega - p\omega_{cs} - \mathbf{k}.\mathbf{V} \qquad (10)$$

and the two resonance conditions (I.1), (I.31') are indeed identical in this regime of strong parallel inhomogeneity.

D. To conclude, we recommend the reader to study a magistral paper of Stix [12], [1] Chapter 17, in which remarkable approximate expressions have been obtained for fundamental ion cyclotron heating: the magnetic-surface-averaged rf power absorbed by the resonant ions, and the steady-state isotropic part of their distribution function.

REFERENCES

An invaluable source of learning on many topics:

1. T. H. Stix, *Waves in Plasmas*, A.I.P., New York (1992).

Geometric optics and ray tracing:

2. C. M. Bender and S. A. Orszag, *Advanced mathematical methods for scientists and engineers*, Mc Graw Hill, New York (1984).
3. I.B. Bernstein, *Phys. Fluids* **18**, 320 (1975).
4. V.P. Bhatnagar, R. Koch, P. Geilfus, R. Kirkpatrick and R.R. Weynants, *Nucl. Fusion* **24**, 955 (1984).
5. D. Van Eester, Ph.D. Dissertation, Katholieke Universiteit Leuven (Belgium), Faculteit der Wetenschappen (1989).
6. D. Van Eester, V.P. Bhatnagar and R. Koch, *Plasma Phys. Control. Fusion* **27**, 1015 (1985)
7. R. Koch, V.P. Bhatnagar, A.M. Messiaen and D. Van Eester, *Comp. Phys. Comm.* **40**, 1 (1986).
8. E. Westerhof, *Plasma Phys. Control. Fusion* **39**, 1015 (1997).
9. E. Westerhof, *24th EPS Conf. Controlled Fusion and Plasma Physics*, Berchtesgaden, Germany (1997), P2.033.

Coupling of full-wave and ray tracing codes:

10. D. Van Eester, *Plasma Phys. Control. Fusion* **34**, 1539 (1992).
11. Y. Petrov, A. Bécoulet and I. Monakhov, Report EUR-CEA-FC-1671, C.E.A., Cadarache, France (September 1999).

A most famous paper on hf wave heating in tokamaks - a must:

12. T. H. Stix, *Nucl. Fusion* **15**, 737 (1975).

Wave-induced stochasticity:

13. D. W. Faulconer and R. Liboff, *Phys. Fluids* **15**, 1831 (1972).
14. M. N. Rosenbluth, *Phys. Rev. Lett.* **29**, 408 (1972).
15. A. J. Lichtenberg and M. A. Lieberman, *Regular and stochastic motion*, Springer-Verlag (1983).

Collisional decorrelation:

16. D. Smithe, P. Colestock, T. Kammash and R. Kashuba, *Phys. Rev. Lett.* **60** 801 (1988).
17. S. V. Kasilov, A. I. Pyatak and K. N. Stepanov, *Nucl. Fusion* **30**, 2467 (1990).
18. D. Van Eester, Y. Louis, P.U. Lamalle and R. Koch, *Phys. Letters* **218A** 70 (1996).

The Hamiltonian approach:

19. A. N. Kaufman, *Phys. Fluids* **15** 1063 (1972); *Erratum in* H. E. Mynick, *J. Plasma Phys.* **39**, 303 (1988).
20. D. J. Gambier and A. Samain, *Nucl. Fusion* **25**, 283 (1985).
21. A. Bécoulet, D.J. Gambier and A. Samain, *Phys.Fluids B* **3**, 137 (1991).
22. D. Van Eester, *J. Plasma Physics* **60**, 627 (1998).

The trajectory integral approach: see e.g.

23. M. Brambilla and T. Krücken, *Nucl. Fusion* **28**, 1813 (1988).
24. P. U. Lamalle, *Phys. Letters* **A175**, 45 (1993).
25. P. U. Lamalle, *Plasma Phys. Control. Fusion* **39**, 1409 (1997); *ibid.* **40**, 465 (1998).
26. S. V. Kasilov, A. I. Pyatak and K. N. Stepanov, in *Reviews of Plasma Physics*, *Vol. 20*, Ed. B. B. Kadomtsev, Consultants Bureau, New York (1997).

See also the discussion in

27. D. Faulconer, *Plasma Phys. Control. Fusion* **29**, 433 (1987).

For a parallel presentation of both formalisms, see

28. R. Koch, P. U. Lamalle and D. Van Eester, *Plasma Phys Control. Fusion* **40**, A191 (1998).
29. D. Van Eester, *Plasma Phys. Control. Fusion* **41**, L23 (1999).

ICRF full-wave codes: see e.g.

30. A. Bécoulet, D. Fraboulet, G. Giruzzi, D. Moreau, B. Saoutic and J. Chinardet, *Phys. Plasmas* **1**, 2908 (1994).
31. M. Brambilla, *Plasma Phys. Control. Fusion* **41**, 1 (1999).

Quasilinear diffusion:

32. I.B. Bernstein and D.C. Baxter, *Phys. Fluids* **24**, 108 (1981).
33. J. Killeen, G.D. Kerbel, M.G. McCoy and A.A. Mirin, *Computational methods for kinetic models of magnetically confined plasmas*, Springer-Verlag (1986).
34. L-G. Eriksson and P. Helander, *Phys. Fluids B* **1**, 308 (1994).

Combined treatments of wave propagation and quasilinear diffusion:

35. J. Hedin, J. Carlsson, T. Hellsten and A. Jaun, *Plasma Phys. Control. Fusion* **40**, 1085 (1998).
36. P.U. Lamalle and D. Van Eester, *2nd Europhysics Topical Conference on RF Heating and Current Drive of Fusion Devices*, Brussels (Belgium) - Europhysics Conf. Abstracts **22A**, 69 (1998).
37. D. Van Eester and P.U. Lamalle, *ibid.* p.73.
38. P.U. Lamalle and D. Van Eester, *17th IAEA Fusion Energy Conference*, Yokohama (1998), p.1639.

ICRF-induced radial drifts:

39. L. Chen, J. Vaclavik and G. W. Hammett, *Nucl. Fusion* **28**, 389 (1988).
40. J. Carlsson, T. Hellsten and J. Hedin, *Phys. Plasmas* **5** 2885 (1998).
41. L-G. Eriksson, M. Mantsinen, D. Borba *et al*, *Phys. Rev. Letters* **81** 1231 (1998).

Theory and modelling of ion cyclotron emission (an important topic not addressed in the text):

42. D. B. Batchelor, E. F. Jaeger and P. L. Colestock, *Phys. Fluids* **B1**, 1174 (1989).
43. D. Fraboulet and A. Bécoulet, *Phys. Plasmas* **4**, 4318 (1997).